

# How to confirm and exclude different models of material properties in the Casimir effect

V M Mostepanenko

Central Astronomical Observatory at Pulkovo of the Russian Academy of Sciences,  
St.Petersburg, 196140, Russia

Institute of Physics, Nanotechnology and Telecommunications, St.Petersburg State  
Polytechnical University, St.Petersburg, 195251, Russia

E-mail: vmostepa@gmail.com

**Abstract.** We formulate a method allowing to confirm or exclude the alternative models of material properties at some definite confidence level in experiments on measuring the Casimir force. The method is based on the consideration of differences between the theoretical and mean measured quantities and the confidence intervals for these differences found at sufficiently high or low confidence probabilities. The developed method is applied to the data of four recent experiments on measuring the gradient of the Casimir force by means of a dynamic atomic force microscope. It is shown that in experiments with Au-Au and Ni-Ni test bodies, where the Drude model approach is excluded at a 95% confidence level, the plasma model approach agrees with the data at higher than 90% confidence. In experiments using an Au sphere interacting with either a Ni plate or a graphene-coated substrate the measurement data agree with the common prediction of the Drude and plasma model approaches and theory using the polarization tensor at 90% and 80% confidence levels, respectively.

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## 1. Introduction

The Casimir effect [1] manifests itself as a force acting between closely spaced material bodies. Similar to the van der Waals force, the Casimir force is caused by the zero-point and thermal fluctuations of the electromagnetic field [2]. The only difference is that the Casimir force acts at larger separation distances and depends on the relativistic retardation. The Casimir effect is a multidisciplinary phenomenon important for condensed matter physics, nanotechnology, elementary particle physics, atomic physics, and for gravitation and cosmology (see the monographs [3–5] and references therein). Recently a lot of experiments on measuring the Casimir force between metallic, semiconductor and dielectric test bodies have been performed (see reviews [6–8] and more recent results [9–23]). The experimental data have been compared with theoretical predictions of the fundamental Lifshitz theory [24] expressing the Casimir force in terms of the frequency-dependent dielectric permittivities of material bodies.

The theory-experiment comparison revealed a fundamental puzzle in Casimir physics. It turned out that for metallic test bodies the theoretical results are in agreement with the measurement data only under a condition that the relaxation properties of conduction electrons are not taken into account in computations [3,6,15,17,25–27]. Technically this means that in calculation of the thermal Casimir force an extrapolation of the measured optical data for the complex index of refraction to zero frequency should be made using the nondissipative plasma model rather than the Drude model taking the relaxation properties of conduction electrons into account. There is one experiment [12] claiming an agreement of the data with the Drude model extrapolation. This experiment, however, measured not the Casimir force, but up to an order of magnitude larger force supposedly originating from the electrostatic patches. The latter force was modeled using the two fitting parameters and subtracted from the data (critical analysis of such a procedure can be found in the literature [29–30]). Another facet of the puzzle is that for the test bodies made of dielectric materials the measured thermal Casimir force is in agreement with the predictions of the Lifshitz theory only if the contribution of free charge carriers to the dielectric permittivity is omitted [3,6,7,11,13,31–34]. It was also shown [3,6,7,35–38] that the Lifshitz theory taking into account the relaxation properties of conduction electrons in metals with perfect crystal lattices or free charge carriers in dielectrics violate the third law of thermodynamics (the Nernst heat theorem).

Attempts to resolve the puzzle in Casimir physics have raised a question on how to make the comparison between experiment and theory more rigorous. At separations below a few hundred nanometers, where measurements of the Casimir force are most precise, the thermal effect predicted by the Drude model does not exceed a few percent of the measured force. As to the thermal effect predicted by the plasma model, at short separation distances it is below the instrumental sensitivity for all materials with exception of graphene [39]. Because of this, up to the present the main attention was paid to statistical procedures allowing to exclude some model of material properties at

sufficiently high confidence level. As was already mentioned, many experiments on the thermal Casimir force resulted in an exclusion of the Drude model for metals and of the role of conduction electrons for dielectrics. In so doing it was sometimes assumed that the confidence levels for the exclusion of the Drude model by the data and for the agreement of the same data with the plasma model are common [25].

In this paper, we discuss how to confirm or exclude some alternative models of material properties when comparing measurements of the thermal Casimir force with theory. We demonstrate that if there are two competing models of material properties, the confidence levels for the exclusion of one of them and for the confirmation of another one are usually different. For this purpose we consider the random quantity  $F'_{\text{diff}}(a)$  equal to differences between theoretical and mean experimental values of the measured gradient of the Casimir force as functions of separation  $a$  between the test bodies. In order to exclude some model of material properties, we calculate the confidence intervals  $[-\Xi(a), \Xi(a)]$  for this quantity at sufficiently high confidence level. Then we verify, whether the values of  $F'_{\text{diff}}(a)$  belong to it. This approach was often used in previous literature [3,6,7,18,25,27,31,32]. To make sure that some model of material properties is in agreement with the data, we determine the confidence intervals for  $F'_{\text{diff}}(a)$  at sufficiently low confidence level. If it is found that the model is not excluded even at this low confidence level, one can conclude that it agrees with the data at a high confidence level complementary to unity. This approach was previously described only briefly and only for a measurement of the Casimir force between two Ni test bodies [18]. Here we apply it to measurements of the gradient of the Casimir force between two Au test bodies [15], between one Au and one Ni test bodies [14], and between an Au test body and a graphene-coated substrate [20]. In all these cases we determine the confidence levels with which the respective theory is in agreement with the measurement data.

The paper is organized as follows. In section 2 we introduce the main notations and briefly formulate the results for Ni-Ni test bodies. In section 3 the case of a Au-coated sphere interacting with a Au-coated plate is considered. Section 4 is devoted to the experiment with a Au-coated sphere interacting with a Ni-coated plate. In section 5 a Au-coated sphere interacting with a graphene-coated substrate is considered. In section 6 the reader will find our conclusions and discussion.

## 2. Selection between models of material properties in Casimir experiment with two Ni test bodies

In all experiments under consideration in this paper, the gradient of the thermal Casimir force  $F'(a)$ , acting between the sphere and the plate spaced at a separation  $a$ , is measured by means of a dynamic atomic force microscope (AFM). There is a standard method to represent the measurement data as crosses centered at points with coordinates  $[a_i, \bar{F}'_{\text{expt}}(a_i)]$ , where  $\bar{F}'_{\text{expt}}(a_i)$  is the mean gradient of the Casimir force measured at a separation  $a_i$ , the length of the horizontal arms is equal to  $2\Delta^{\text{tot}}a_i$  and the height of

the vertical arms is equal to  $2\Delta^{\text{tot}}\bar{F}'_{\text{expt}}(a_i)$  [3]. Here, the total errors (i.e., the random and systematic combined),  $\Delta^{\text{tot}}a_i$  and  $\Delta^{\text{tot}}\bar{F}'_{\text{expt}}(a_i)$ , are meant to be determined at some common confidence level. If the theoretical band does not overlap with the majority of crosses within some range of separations, it is said that this theory (theoretical model) is excluded by the data at a given confidence level. In doing so, the width of the theoretical band is equal to twice the theoretical error,  $\Delta^{\text{tot}}F'_{\text{theor}}(a)$  with which theoretical values of the force gradient are computed [3, 6]. Although this method of experiment-theory comparison allows to determine the confidence level of exclusion of some model by the data, it is somewhat difficult to quantitatively characterize the measure of agreement when there is a partial overlap between the experimental crosses and the theoretical band.

Another method to compare experiment with theory is based on the consideration of the random quantity

$$F'_{\text{diff}}(a) = F'_{\text{theor}}(a) - \bar{F}'_{\text{expt}}(a). \quad (1)$$

Here, the theoretical force gradients are computed with the help of the Lifshitz theory using some model of dielectric response and taking into account the surface roughness. The confidence interval for this quantity at a given confidence level  $\beta$  can be expressed via the total experimental and theoretical errors. The conservative estimation for the halfwidth of this interval at a separation  $a_i$  is given by [3, 6]

$$\Xi_{F'_{\text{diff}}}^{\beta}(a_i) = \min \left\{ \Delta^{\text{tot}}F'_{\text{theor}}(a_i) + \Delta^{\text{tot}}\bar{F}'_{\text{expt}}(a_i), \right. \\ \left. k_{\beta} \sqrt{[\Delta^{\text{tot}}F'_{\text{theor}}(a_i)]^2 + [\Delta^{\text{tot}}\bar{F}'_{\text{expt}}(a_i)]^2} \right\}. \quad (2)$$

Here,  $k_{\beta}$  is a tabulated coefficient from the composition law of two uniform distributions [41]. As an example, for  $\beta = 0.95$  (a 95% confidence level)  $k_{\beta} = 1.1$ . If for some theoretical model of material properties more than  $1 - \beta$  percent of the differences  $F'_{\text{diff}}$  are outside the confidence interval

$$[-\Xi_{F'_{\text{diff}}}^{\beta}(a_i), \Xi_{F'_{\text{diff}}}^{\beta}(a_i)] \quad (3)$$

for all  $a_i$  belonging to some interval  $[a_{\min}, a_{\max}]$ , one can conclude that this model is experimentally *excluded* for separations from  $a_{\min}$  to  $a_{\max}$  at a confidence level  $\beta$ . Alternatively, a model of material properties, for which no fewer than  $\beta$  percent of the differences  $F'_{\text{diff}}$  belong to the confidence interval (3) within any separation subinterval from  $a_{\min}$  to  $a_{\max}$ , is *consistent* with the measurement data within this confidence interval. In fact, this comparison method is close in spirit to the so-called “p-value” in statistics [40], except in our case it is applied over the aggregate of the measurements done at different separations. Note, however, that the method of p-value is most often applied for the verification of the null hypothesis in psychological, medical, and economy research which are more empirical than physical research. The latter is often based on the well-established fundamental theory.

Let us assume that some theoretical model is *consistent* with the data within the confidence interval defined at some high confidence level  $\beta$ . It should be particularly

emphasized that this does not mean that the model is *confirmed* by the data, or, synonymously, agrees with the data at the same high confidence. Really, the higher is the confidence probability  $\beta$ , the wider is the confidence interval (3). For example, if some theoretical approach is excluded at a 95% confidence level, this means that the model agrees with the data at only 5% probability. At this point, a terminological note is pertinent. In fact it is common to speak about confirmation of physical theories by the measurement data. It should be remembered, however, that this “confirmation” is not absolute, so that some theory can be confirmed at rather high confidence level  $\beta_1$  (i.e., not excluded at a confidence level  $1 - \beta_1$ ), but excluded at a confidence level  $\beta_2 < 1 - \beta_1$ . Keeping this in mind and to avoid confusion, we will subsequently speak about “agreement of the model with the data at some confidence level” as a synonym for “confirmation of the model by the data at some confidence level”.

Let us consider in more detail the difference between the concepts of *consistency* and *confirmation*. If almost 100% of the differences (1) belong to the confidence interval (3) defined at a confidence level  $\beta = 0.95$  (i.e., the model under consideration is *consistent* with the data within the 95% confidence interval), this does not mean a high degree of *agreement*. Just on the contrary, belonging of the differences (1) to a sufficiently wide interval (3) may mean rather poor agreement between the theoretical and experimental force gradients. Specifically, some theoretical model consistent with the data within a 95% confidence interval might be excluded by the same data at a 67% confidence level. It can be concluded that the degree of *agreement* of some theoretical model increases *if a sufficient fraction of the differences (1) is found inside the confidence interval (3) defined at rather low confidence level* (e.g.,  $\beta = 0.1$ ). For instance, let one finds more than 10% of the differences (1) inside such a confidence interval (3) within any separation subinterval. This would mean that the theoretical model is not excluded by the measurement data even at a 10% confidence level, or, equivalently, that this model is in agreement with the data at a higher than 90% confidence level.

To make the above formulations more accessible, we illustrate them by the data of experiment on measuring the gradient of the Casimir force between a Ni-coated sphere of  $R = 61.71 \mu\text{m}$  radius and a Ni-coated plate by means of dynamic AFM [17, 18]. The mean measurement data for the gradient of the Casimir force at  $T = 300 \text{ K}$ ,  $\bar{F}'_{\text{expt}}(a_i)$ , were compared with the theoretical predictions of the Lifshitz theory [3, 24, 42] describing the dielectric properties of Ni by means of the frequency-dependent dielectric permittivity  $\varepsilon^{\text{Ni}}(\omega)$ . The magnetic properties of Ni were described by the Debye formula [18] for the frequency-dependent magnetic permeability  $\mu(\omega)$ . For ferromagnetic materials  $\mu(i\xi)$  becomes equal to unity at  $\xi > 10^5 \text{ Hz}$ , i.e., starting from frequencies much smaller than the first Matsubara frequency at room temperature. From this it follows that the magnetic Casimir interaction is determined by the static magnetic permeability  $\mu^{\text{Ni}}(0) = 110$  (quick decrease of  $\mu(i\xi)$  to unity with increasing  $\xi$  and its role in the magnetic Casimir effect was discussed in [43]).

The dielectric properties of Ni were described using two different models. Within the first model, the dielectric permittivity at the imaginary Matsubara frequencies was

obtained using the standard Kramers-Kronig relation from

$$\text{Im } \varepsilon(\omega) = 2n_1(\omega)n_2(\omega), \quad (4)$$

where  $n_1(\omega)$  and  $n_2(\omega)$  are the real and imaginary parts of the complex index of refraction, respectively, measured over a wide frequency region [44]. Below the minimum frequency, where the optical data are not available,  $\text{Im } \varepsilon(\omega)$  was extrapolated down to zero frequency by means of the Drude model (the so-called *Drude model approach*)

$$\varepsilon_D(\omega) = 1 - \frac{\omega_p^2}{\omega[\omega + i\gamma(T)]}. \quad (5)$$

Here, for Ni the plasma frequency  $\omega_p = 4.89$  eV and the relaxation parameter at room temperature  $\gamma(T) = 0.0436$  eV [44, 45] were used.

Within the second model, the optical data with subtracted contribution of conduction electrons are used to find the parameters of oscillators describing the bound (core) electrons. Then the dielectric permittivity is represented in the form

$$\varepsilon_{gp}(\omega) = 1 - \frac{\omega_p^2}{\omega^2} + \sum_{j=1}^K \frac{g_j}{\omega_j^2 - \omega^2 - i\gamma_j\omega}, \quad (6)$$

where  $K$  is the number of oscillators,  $g_j$  are the oscillator strengths, and  $\gamma_j$  are the relaxation frequencies (this is the so-called *generalized plasma-like model*). Thus, in the second model of dielectric properties the optical data should be rid off the contribution of free electrons and extrapolated to zero frequency by means of the nondissipative plasma model

$$\varepsilon_p(\omega) = 1 - \frac{\omega_p^2}{\omega^2}, \quad (7)$$

describing the plasma of free electrons (the so-called *plasma model approach*). Equivalently, the same results can be obtained directly from the imaginary part of the dielectric permittivity (4), where the optical data are rid off the contribution of free electrons, by using the generalized Kramers-Kronig relation applicable to dielectric functions having the second-order pole at zero frequency [3, 6, 46].

It is necessary to stress that at low (quasistatic) frequencies the Maxwell equations lead to the dielectric permittivity of the Drude-type (5) which is inverse proportional to the first power of frequency [47]. As to the plasma model (7), it is applicable in the region of high (infrared) frequencies satisfying the condition  $\omega \gg \gamma(T)$ . Because of this, an exclusion of the Drude model approach and consistency with the plasma model approach in the most of experiments on measuring the Casimir force discussed in section 1 is considered as a puzzle.

Now let us return to the experiment [17, 18] dealing with two Ni test bodies and consider the quantitative measures of exclusion and agreement for different models of material preperities. In figure 1(a,b) we present as dots the differences of the gradients of the Casimir force (1) as functions of separation computed using the Drude model (the upper set of gray dots) and the plasma model (the lower set of black dots) approaches. The upper and lower solid lines indicate the borders of the confidence intervals (3) as

functions of separation computed using (2) at (a) 95% and (b) 67% confidence levels. Note that the borders of the confidence intervals very slightly depend on the theoretical approach used and are mostly determined by the chosen confidence level. As is seen in figure 1(a,b), the Drude model approach is excluded by the data over the separation region from 223 to 345 nm at a 95% confidence level and from 223 to 420 nm at a 67% confidence level, respectively. Here, not a bit more than 5% (respectively, 33%) of all dots are outside the confidence intervals, as required for the exclusion of the model, but almost 100% do not belong to the confidence intervals. These are rather strong evidences to believe that this approach is not adequate.

From figure 1(a,b) it is also seen that the plasma model approach is consistent with data at both 95% and 67% confidence levels over the entire measurement range from 223 to 550 nm. As explained above, however, the consistency at rather high confidences (i.e., for rather wide confidence intervals) does not automatically mean that the approach agrees with the data at a high probability. To find the measure of agreement between the plasma model approach and the data, it is necessary to consider sufficiently low confidence probability  $\beta$  (i.e., narrow confidence interval) such that at slightly smaller  $\beta$  this approach is already excluded by the data. In figure 2 we plot as black dots the same differences (1) computed using the plasma model approach as are shown in figure 1(a,b). The upper and lower solid lines indicate the borders of the confidence intervals found at a 10% confidence level. As is seen in figure 2, more than 10% of all dots within any separation subinterval are inside the 10% confidence intervals. This allows one to conclude that the plasma model approach is not excluded by the data even at sufficiently low 10% confidence level, i.e., this approach agrees with the data at no less than 90% confidence.

### 3. Selection between models of material properties in the Casimir experiment with two Au test bodies

Here, we consider the choice between the plasma and the Drude model approaches in the experiment on measuring the gradient of the Casimir force between two Au-coated surfaces of a sphere of  $R = 41.3 \mu\text{m}$  radius and a plate by means of dynamic AFM [15]. In this experiment the Drude model approach was excluded at a 67% confidence level over the separation region from 235 to 420 nm using a standard method of theory-experiment comparison described in the beginning of section 2. Later on, using the statistical method applied here, it was shown that the data of this experiment exclude the Drude model approach at a higher 95% confidence level over the separation region from 235 to 330 nm. The plasma model approach was found to be consistent with the data at both confidence levels. However, the confidence level for its agreement with the data was not found.

Before presenting the resolution of this question, it is pertinent to note that recently a distinction between the predictions of the Drude and plasma model approaches for two bodies made of nonmagnetic metal (such as Au) has been questioned [48]. Specifically, it

was claimed [48] that for a nondissipative plasma model the Lifshitz formula in the form of summation over the Matsubara frequencies undergoes a modification. As a result, the discontinuity between the predictions of the Drude and plasma model approaches disappears [48]. According to [48], the physical reason for this is the contribution of the Foucault currents which plays a role even in the limit of vanishing dissipation, in contrast to commonly accepted views.

In this connection we underline that the above claims of [48] are based on a terminological confusion and do not contain any new physics. The nondissipative plasma model is defined in [48] not in accordance to the conventional definition (7), but as the limiting case of the Drude model (5) when  $\gamma \rightarrow 0$ . This would be really correct if only nonzero frequencies  $\omega \neq 0$  were considered. However, [48] employs this definition at all frequencies including  $\omega = 0$ . Such an employment is mathematically unjustified because

$$\begin{aligned} \lim_{\gamma \rightarrow 0} \varepsilon_D(\omega) &= 1 - \lim_{\gamma \rightarrow 0} \frac{\omega_p^2}{\omega^2 + \gamma^2} + i \frac{\omega_p^2}{\omega} \lim_{\gamma \rightarrow 0} \frac{\gamma}{\omega^2 + \gamma^2} \\ &= 1 - \frac{\omega_p^2}{\omega^2} + i\pi \frac{\omega_p^2}{\omega} \delta(\omega). \end{aligned} \quad (8)$$

It is seen that the limiting case (8) of the Drude model (5) when  $\gamma \rightarrow 0$  coincides with the plasma model (7) only at  $\omega \neq 0$ . At  $\omega = 0$  it contains an infinitely large imaginary part, i.e., is not a nondissipative dielectric permittivity. Thus, the expression (8) does not coincide with the plasma model (7) and cannot be called “the plasma model”. Note also that in accordance to previously published results [48–50] it is just the contribution of the Foucault currents which determines a difference in the theoretical predictions of the Drude and plasma model approaches.

Now we return to the measurement data of the experiment [15] measuring the gradient of the Casimir force between two Au-coated surfaces of a sphere and a plate. In figure 3(a,b) we present as dots the differences (1) at different separations found using the Drude model (the lower sets of gray dots) and the plasma model (the upper sets of black dots) approaches. The pairs of solid lines indicate the borders of the confidence intervals (3) at different separations found using (2) at (a) 95% and (b) 67% confidence levels. The comparison with figure 1(a,b) shows that for Ni and Au the differences  $F'_{\text{diff}}$  computed using the Drude model approach have the opposite signs. This was used [17, 18] to exclude the role of any unaccounted systematic effect in the measurement data. As was mentioned in the beginning of this section, the Drude model approach is excluded by the data, whereas the plasma model approach is experimentally consistent.

To characterize the measure of agreement between the plasma model approach and the measurement data, in figure 4 we plot as black dots the same differences as shown by the black dots in figure 3(a,b). The pairs of solid lines in figure 4 indicate the borders of the confidence intervals found at a 10% confidence level. It can be seen that more than 10% of all dots within any separation subinterval belong to the confidence intervals. Thus, the plasma model approach is not excluded by the data at a 10% confidence level. As a result, one can say that in the experiment with two Au surfaces the plasma model approach agrees with the data at no less than 90% confidence level. This conclusion is



the same as was done in previous section with respect to the experiment with two Ni surfaces.

#### 4. Agreement with the models of material properties in the Casimir experiment with Au-Ni test bodies

In the experiment [14] the gradient of the Casimir force was measured between a Au-coated sphere of  $R = 64.1 \mu\text{m}$  radius and a Ni-coated plate by means of dynamic AFM. For this configuration within the used separation region from 220 to 500 nm the theoretical predictions of the Drude and plasma model approaches almost coincide. Thus, the magnitudes of relative differences in the predictions of both models at separations 220, 300, 400, and 500 nm are only 0.5%, 0.2%, 0.4%, and 1.2%, respectively [14]. This is below the instrumental sensitivity at respective separations. At larger separations the relative differences between the two approaches are much larger. For example, at separations of 3 and  $5 \mu\text{m}$  they are equal to 31.1% and 42.8%, respectively [14]. Because of this, short separation measurements of the gradient of the Casimir force between a Au-coated sphere and a Ni-coated plate provide unique opportunity of testing all experimental procedures for the presence of any systematic effect which might influence theory-experiment comparison in other measurements.

Now we determine at what confidence level the used models of material properties agree with the experimental data of the experiment [14]. In figure 5 the differences of the gradients of the Casimir force (1) at different experimental separations using (a) the plasma model approach and (b) the Drude model approach are shown as the black and gray dots, respectively. The pairs of solid lines indicate the borders of the confidence intervals calculated at a 67% confidence level. As is seen in figure 5, both the plasma and the Drude model approaches are experimentally consistent at a 67% confidence level. This is not surprising taking into consideration that within the experimental separation region both these approaches predict almost the same gradients of the Casimir force.

Similar differences of the gradients of the Casimir force found using (a) the plasma model approach and (b) the Drude model approach are plotted in figure 6, where the two solid lines show the borders of a 10% confidence. In both cases more than 10% of the dots belong to the confidence interval within any separation subinterval. This means that in the case of Au-Ni interacting bodies both the plasma and the Drude model approaches agree with the measurement data at no less than 90% confidence level. The probability that their common prediction is incorrect is less than 10%. This invalidates an assumption [52] proposed for the configuration of Au-Au test bodies that there may be some unaccounted systematic effect (supposedly originating from large surface patches) which superimposes on the Casimir force and effectively brings the data in agreement with the plasma model approach. It is true that for Au-Au surfaces, where the two theoretical approaches predict different gradients of the Casimir force, this is formally possible. However, for Au-Ni test bodies any unaccounted systematic effect would bring the measurement data in disagreement with both the Drude and the plasma model

approaches. Recently the voltage distribution due to surface patches on the planar Au samples used in Casimir force measurements was directly measured by means of Kelvin probe force microscopy [53]. It was shown that the gradient of respective additional force is more than one order of magnitude smaller and has a stronger separation dependence than the difference in theoretical predictions of the plasma and Drude model approaches.

## 5. Agreement with a model of material properties in the Casimir experiment with graphene-coated substrate

Graphene is a two-dimensional sheet of carbon atoms having unusual electrical, mechanical and optical properties [54]. It was shown [55] that for graphene the thermal effect in the Casimir interaction becomes dominant at much shorter separation than for ordinary materials, either dielectric or metallic. This is the reason why graphene is of much promise for experiments on measuring the Casimir force. The material properties of graphene cannot be described by the dielectric permittivity depending only on the frequency. In the framework of the Dirac model [54] the fundamental description of the response of graphene on the electromagnetic field is described by the polarization tensor [56]. The equivalent description can be formulated in terms of the density-density correlation functions and nonlocal dielectric permittivities depending on both the frequency and the wave vector [57].

The first measurement of the Casimir interaction in the configuration including a graphene sheet was performed very recently [20]. In this experiment the gradient of the Casimir force between a Au-coated sphere of  $R = 54.1 \mu\text{m}$  radius and a graphene sheet deposited on a  $\text{SiO}_2$  film covering a Si plate has been measured. Theoretical description of the Casimir interaction for layered structures, where some layers are described by the frequency-dependent dielectric permittivities and some other by the polarization tensor, was elaborated in 2014 [58]. Using this theory, the measurement data of [20] were shown to be in agreement with the predictions of the Lifshitz theory [58], but only the first method of theory-experiment comparison considered in section 2 has been used. Here, we consider the differences between theoretical and mean experimental gradients of the Casimir force and determine more rigorously the measure of agreement between experiment and theory.

In figure 7 the differences between the gradients of theoretical and mean experimental Casimir forces are shown by dots for (a) the first and (b) the second graphene-coated sample as functions of separation. The two solid lines indicate the borders of the 67% confidence intervals. From figure 7 one can only conclude that the used theory is consistent with the measurement data for both samples with no indication of any quantitative measure of agreement.

To make the comparison more informative, in figure 8 we plot the same differences of the gradients of the Casimir force as dots, but plot as two lines the borders of the 10% confidence intervals. From figure 8 one can see that there are such subintervals in the entire separation range from 224 to 500 nm where less than 10% of all dots

belonging to these subintervals belong to the confidence intervals. Thus, unlike the examples, considered in sections 2–4, for a graphene-coated substrate the favorite theory is excluded by the data at a 10% confidence level.

To find the measure of agreement between experiment and theory for graphene, in figure 9 we again plot the same differences as dots but indicate by the two lines the borders of the 20% confidence intervals. From figure 9 it is seen that for both samples in any separation subinterval more than 20% of all dots belong to the respective confidence intervals. This means that the theory of the Casimir interaction with graphene-coated substrates is not excluded by the data at a 20% confidence level. Equivalently, one can conclude that this theory agrees with the data at no less than 80% confidence level, or that it is correct with at least 80% probability. If to compare with experiments using metallic surfaces, this is a lower level of confidence. This is explained by the lack of information about the properties of graphene sample needed to make the theory more precise (such as the character of impurities, the value of the gap parameter etc. [20]).

## 6. Conclusions and discussion

In the foregoing we have considered the statistical methods of comparison between experiment and theory allowing to find an exclusion or agreement of some models of material properties with the data of experiments on measuring the Casimir interaction. This problem is gaining in importance in the context of the puzzle in theory-experiment comparison in the Casimir physics discussed in section 1. In previous literature the main attention was paid to an exclusion of one of the models at some high confidence level whereas the measures of agreement with the data of experimentally consistent models were not determined.

In this paper we have shown that the probability of agreement of some model of material properties with the data can be found by considering the differences between theoretical and mean measured quantities and the confidence intervals for these differences determined at rather low (10% or 20%) confidence levels. The criterium is introduced that if more than 10% or 20% differences are found inside the respective confidence intervals one can state that the theoretical model is not excluded at these confidence levels and, thus, agrees with the data at no less than the complementary to unity confidence levels equal to 90% or 80%, respectively.

The suggested criterium was applied to the experimental data of four experiments on measuring the gradient of the Casimir force by means of dynamic AFM and two theoretical approaches proposed in the literature for metallic test bodies. It was shown that in the experiments using Au-Au and Ni-Ni test bodies [15, 17, 18] the Drude model approach to calculation of the Casimir interaction is experimentally excluded at a 95% confidence level over some ranges of separations. The plasma model approach agrees with the measurement data of these experiments at no less than 90% confidence level. The same analysis was performed in application to measurements of the gradient of the Casimir force between a Au-coated sphere and a Ni-coated plate [14]. Here, the Drude

and the plasma model approaches lead to almost the same predictions which agree with the data at a 90% confidence level over the entire measurement range. This result allows to exclude the role of any unaccounted systematic effect (such as patch potentials) in theory-experiment comparison. Finally, we have shown that the experimental data of experiment [20] performed in the configuration of a Au-coated sphere and a graphene-coated substrate are in agreement with theory using the polarization tensor of graphene at the 80% confidence level.

The proposed method of comparison between experiment and theory can be used in other precise experiments on measurement of the Casimir force.

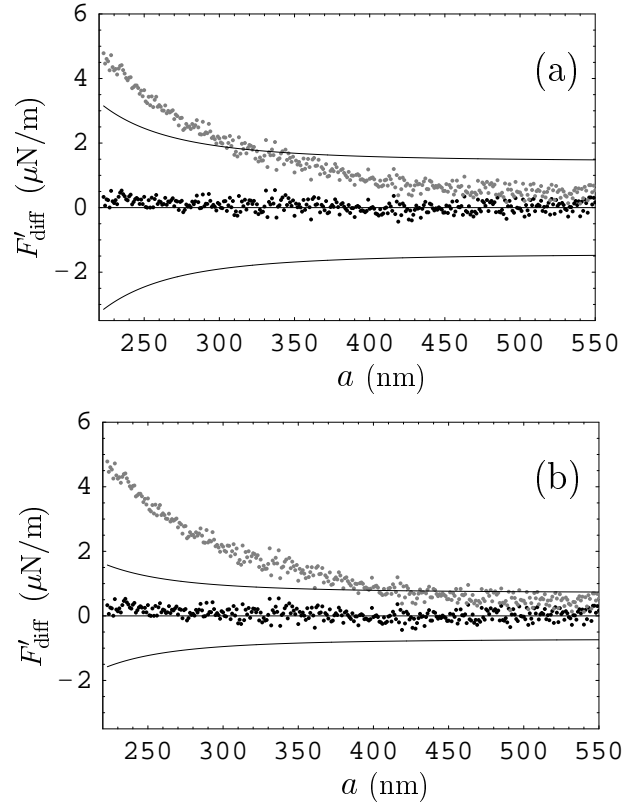
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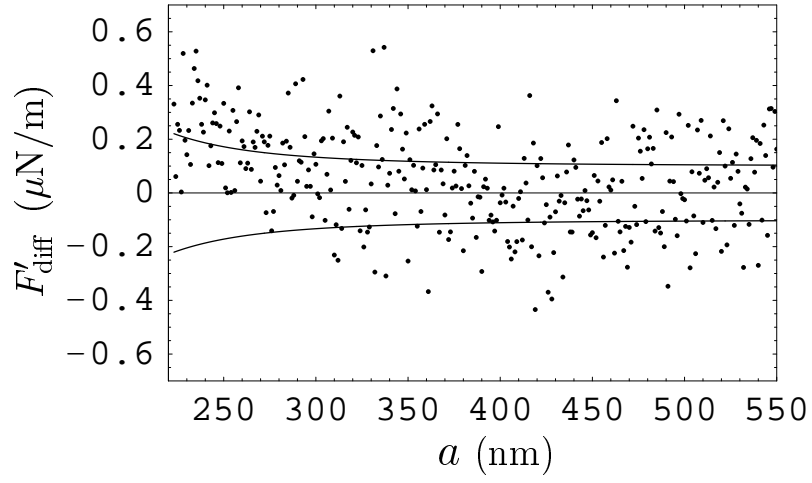
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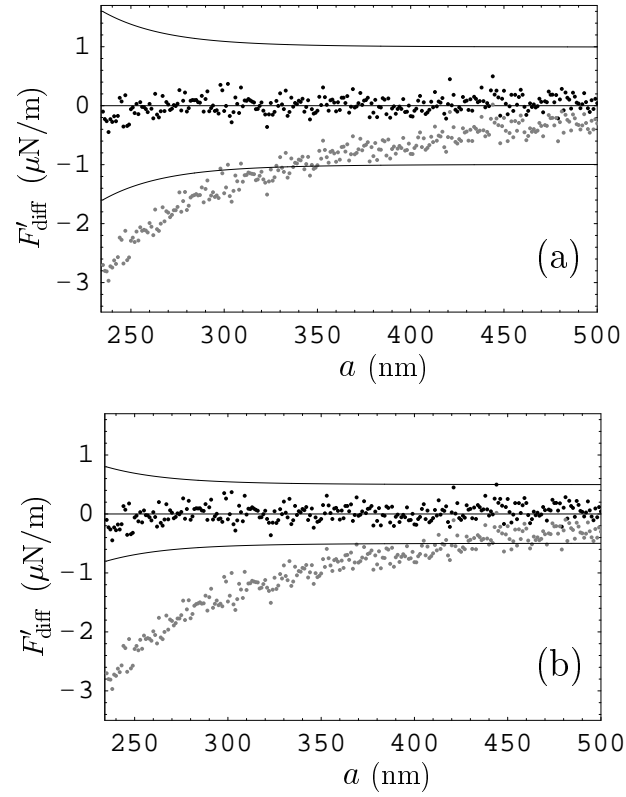
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**Figure 1.** Differences between the theoretical and mean experimental gradients of the Casimir force for Ni-Ni surfaces using the Drude and plasma model approaches are shown by the gray and black dots, respectively [18]. The solid lines indicate the borders of the (a) 95% and (b) 67% confidence intervals.

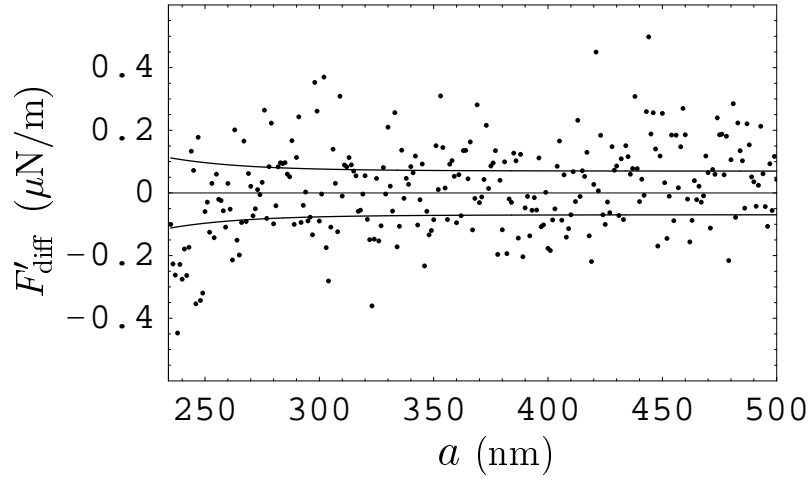


**Figure 2.** Differences between the theoretical and mean experimental gradients of the Casimir force for Ni-Ni surfaces found using the plasma model approach are shown by the black dots [18]. The solid lines indicate the borders of the 10% confidence intervals.

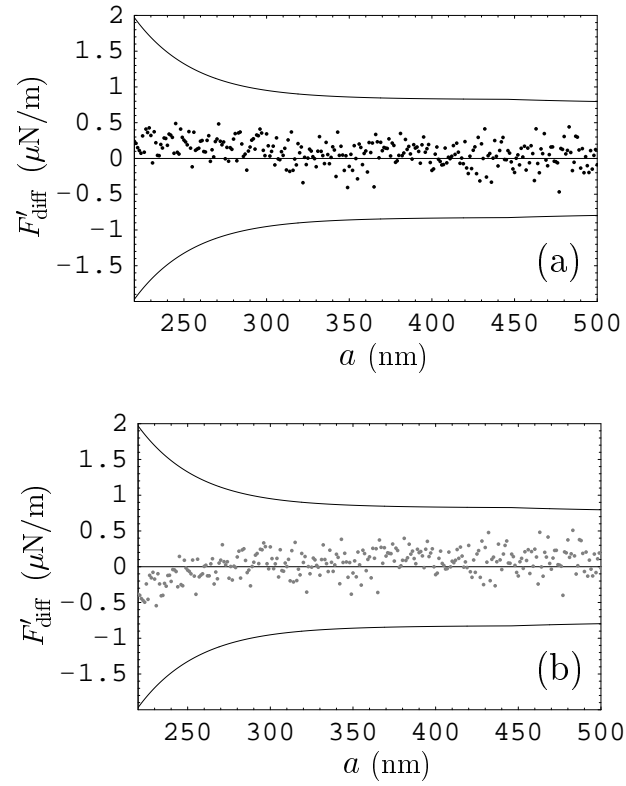


**Figure 3.** Differences between the theoretical and mean experimental gradients of the Casimir force for Au-Au surfaces using the Drude and plasma model approaches are shown by the gray and black dots, respectively. The solid lines indicate the borders of the (a) 95% and (b) 67% confidence intervals.

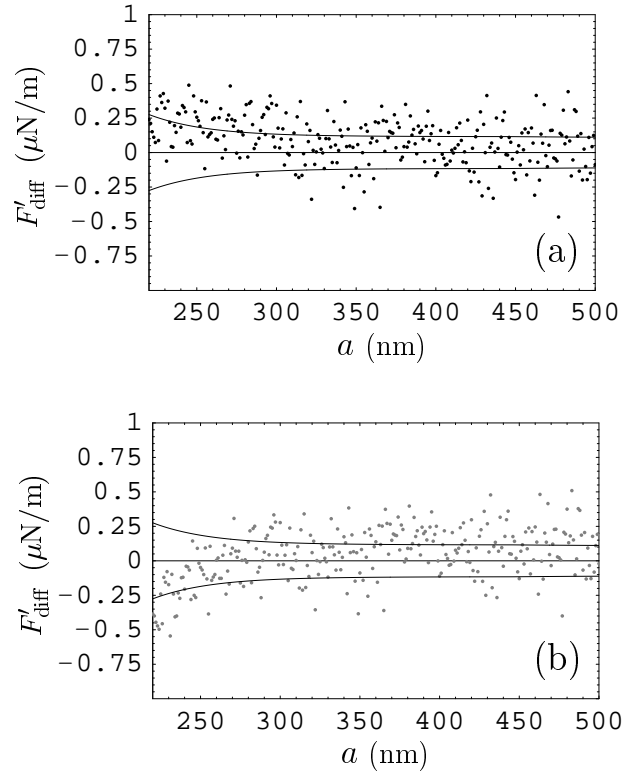




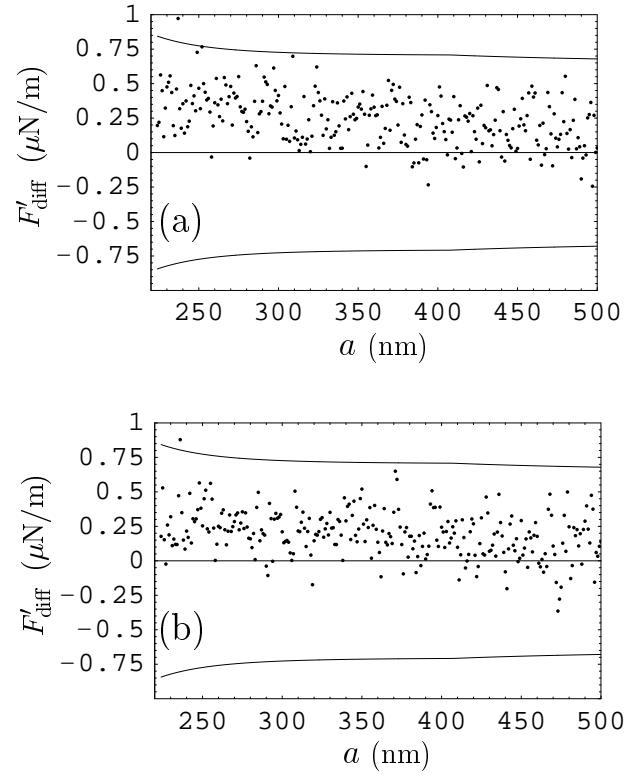
**Figure 4.** Differences between the theoretical and mean experimental gradients of the Casimir force for Au-Au surfaces found using the plasma model approach are shown by the black dots. The solid lines indicate the borders of the 10% confidence intervals.



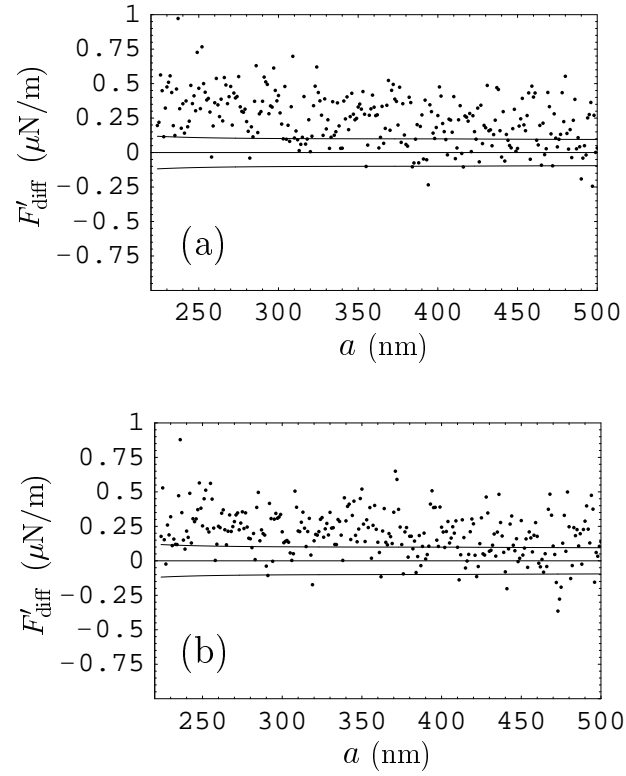
**Figure 5.** Differences between the theoretical and mean experimental gradients of the Casimir force for Au-Ni surfaces found using (a) the plasma and (b) the Drude model approaches are shown by the black and gray dots, respectively. The solid lines indicate the borders of the 67% confidence intervals.



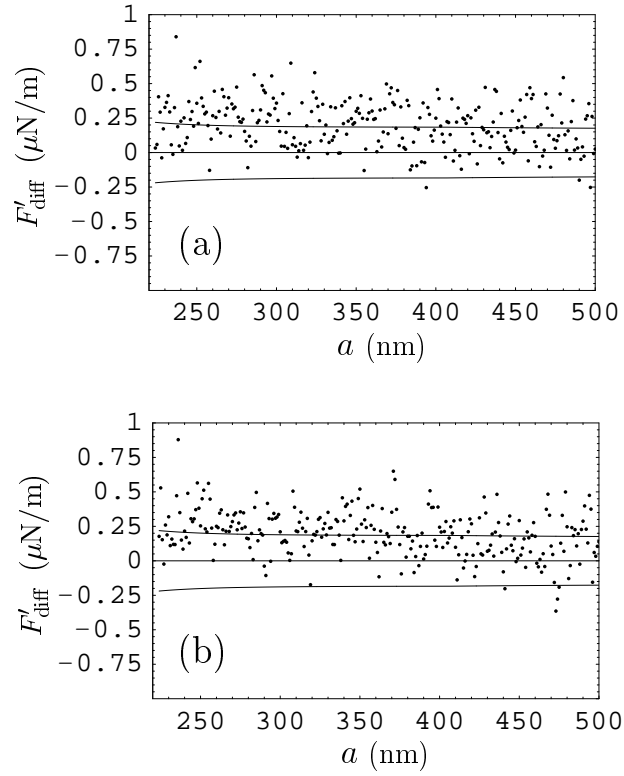
**Figure 6.** Differences between the theoretical and mean experimental gradients of the Casimir force for Au-Ni surfaces found using (a) the plasma and (b) the Drude model approaches are shown by the black and gray dots, respectively. The solid lines indicate the borders of the 10% confidence intervals.



**Figure 7.** Differences between the theoretical and mean experimental gradients of the Casimir force for the interaction of a Au-coated sphere with a graphene-coated substrate are shown as dots for (a) the first and (b) the second graphene-coated sample. The solid lines indicate the borders of the 67% confidence intervals.



**Figure 8.** Differences between the theoretical and mean experimental gradients of the Casimir force for the interaction of a Au-coated sphere with a graphene-coated substrate are shown as dots for (a) the first and (b) the second graphene-coated sample. The solid lines indicate the borders of the 10% confidence intervals.



**Figure 9.** Differences between the theoretical and mean experimental gradients of the Casimir force for the interaction of a Au-coated sphere with a graphene-coated substrate are shown as dots for (a) the first and (b) the second graphene-coated sample. The solid lines indicate the borders of the 20% confidence intervals.